Endogenous Cartel Formation with Differentiated Products

Tyra Merker

PhD-lunch, February 8th 2018

Outline

1. Motivation and literature review
2. The model
    - Timing of the game
    - The price game with differentiated products
3. Equilibrium of the repeated price game
    - The static and subgame perfect Nash equilibria
    - Equilibrium prices with collusion
    - Incentive compatibility and cartel break-down
4. Equilibrium of the cartel formation stage – with symmetric firms
    - Cartel stability
    - Incentive compatibility

Motivation – what drives cartel formation?

- Cartels are harmful to consumers and economic efficiency.
  - Understanding how cartels form may facilitate cartel detection.
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- Research questions:
  - (i) Under what industry conditions can we expect a cartel to be industry-wide?
  - (ii) What market characteristics matter the most for the degree of cartelization?
  - (iii) Which firms are most likely to form a cartel?

Literature review

- Literature on **cartel stability** from the 70's and 80's foundation for models of coalition formation in environmental economics.
  - Selten, R. (1973) 'A simple model of imperfect competition, where 4 are few and 6 are many', *International Journal of Game Theory* **2**(1), 141–201.
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- To my knowledge, no dynamic models of endogenous cartel formation with *differentiated products* exist.
  - Selten (1973) assesses profitability of mergers with differentiated price competition. Shows that mergers are profitable, as prices are strategic complements.

Back to my contribution

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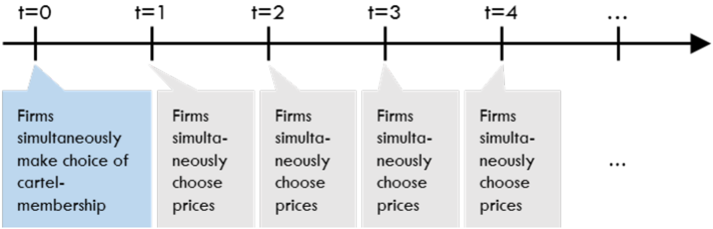
  - Linear demand
  - Linear costs
  - Symmetric firms
  - No renegotiation/recartelization after cartel breakdown
  - ...

  ... and even the simple model gets quite ugly...

Outline

Timing of the game



- Firms first make a decision of whether or not to joint the cartel
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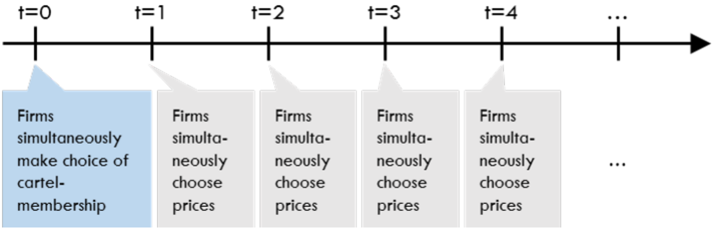

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  - Firms set prices simultaneously each period.
  - Cartel members will set prices differently than non-members.
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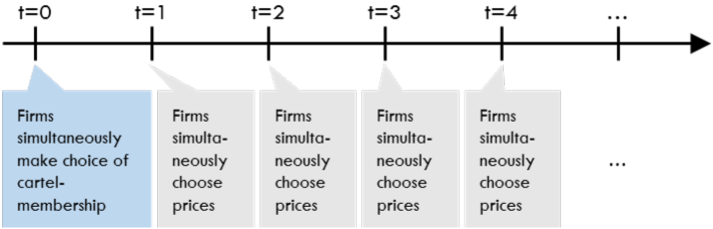

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- Solution strategy:
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Outline

The price game with differentiated products

- Let $\mathcal{N} = \{1, 2, ..., N\}$ denote a set of firms in a market.
- Let $Q_i(p_i, p_{-i})$, for $i \in \mathcal{N}$, denote the demand for firm i's product, where p_i denotes firm i's price and p_{-i} denotes the vector of prices set by i's competitors.

Assumption 1 Properties of the demand function

Assume that demand functions are twice and continuously differentiable, and that for all $i, j \in \mathcal{N}$:

(i) $\frac{\partial Q_i(p_i, p_{-i})}{\partial p_i} \leq 0$

(ii) $\frac{\partial Q_i(p_i, p_{-i})}{\partial p_j} \geq 0$, for $i \neq j$

(iii) $\frac{(p_i - c_i)}{2} \frac{\partial^2 Q_i}{\partial p_i^2} \leq -\frac{\partial Q_i}{\partial p_i}$

- Demand for a firm's product is *de*creasing in its own price.
- Demand for a firm's product is increasing when a competitor raises its price.
- Part (iii) ensures concavity of the firms' profit functions.

The price game with differentiated products

Assumption 2 Linear demand

Assume that linear demand for the product of firm $i \in \mathcal{N}$ takes the form

$$Q_i(p_i, p_{-i}) = A + \sum_{j \in \mathcal{N}} d_{ij} p_j.$$

Furthermore, assume that

(i) a firm's demand increases in its competitors' prices: $d_{ij} \geq 0$ if $j \neq i$

(ii) a firm's demand decreases in its own price: $d_{ij} \leq 0$ if $i = j$

(iii) the total increase in demand experienced by a firm's competitors cannot be greater than the direct demand decline by that firm: $\sum_{j \neq i} d_{ji} \leq -d_{ii}$.

- The first two conditions are analogous to those in Assumption 1.
- Note that if we assume that there is no exit by consumers following a price increase, the last condition holds with equality.

The price game with differentiated products

- Firms set prices simultaneously in each stage-game.
- Each firm $i \in \mathcal{N}$ chooses p_i to maximize its profit $\Pi_i(p_i, p_{-i}) = (p_i - c_i)Q_i(p_i, p_{-i})$, where c_i denotes the constant(!) marginal cost of production.

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- In choosing price p_i, firm i exerts an externality on the other firms in the market. Industry profit can be characterized as $\Pi = \sum_{i \in \mathcal{N}} (p_i - c_i)Q_i(p_i, p_{-i})$. Thus, a change in one firm's price changes industry profit by:

$$\frac{\partial \Pi}{\partial p_i} = \underbrace{(p_i - c_i)\frac{\partial Q_i}{\partial p_i} + Q_i}_{\text{firm i's optimality condition (FOC)}} + \underbrace{\sum_{j \in \mathcal{N}, j \neq i} (p_j - c_j)\frac{\partial Q_j}{\partial p_i}}_{\text{sum of externalities on other firms in market}}$$

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- Free-riding incentive: as $\frac{\partial Q_i}{\partial p_j} \geq 0$, firm i benefits if its competitors collude, as their increased prices lead to higher demand for firm i. However, if it free-rides, it does not need to limit its own price.
- Thus the price-setting stage game constitutes a prisoner's dilemma. $\Rightarrow$ Firms have an incentive to collude.

Solution strategy

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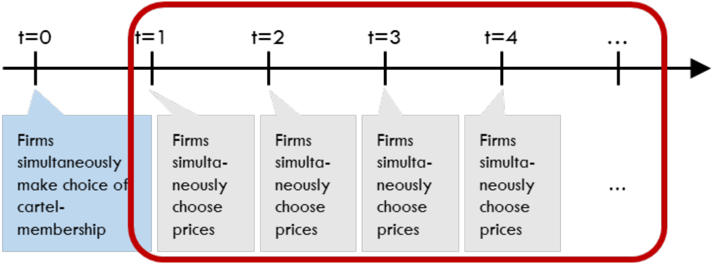

Outline

The static and subgame perfect Nash equilibria

- Profit for each firm $i \in \mathcal{N}$: $\Pi_i(p_i, p_{-i}) = (p_i - c_i) Q_i(p_i, p_{-i})$
- FOCs in the general case (for each $i \in \mathcal{N}$):

$$Q_i(p_i, p_{-i}) + \frac{\partial Q_i(p_i, p_{-i})}{\partial p_i}(p_i - c_i) = 0$$

- FOCs with linear demand (for each $i \in \mathcal{N}$):

$$2d_{ii} p_i + \sum_{j \neq i, j \in \mathcal{N}} d_{ij} p_j = d_{ii} c_i - A$$

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- With linear demand (and quadratic profit functions), FOCs are linear. This yields the analytical solution:

$$\mathbf{p}^{NE} = (\mathbf{D}^{NE})^{-1}\mathbf{B}^{NE} \qquad \text{, where}$$

$$\mathbf{D}^{NE} = \begin{bmatrix} 2d_{11} & d_{12} & \dots & d_{1n} \\ d_{21} & 2d_{22} & & \\ \vdots & & \ddots & \vdots \\ d_{n1} & \dots & & 2d_{nn} \end{bmatrix}, \text{ and } \qquad \mathbf{B}^{NE} = \begin{bmatrix} d_{11}c_1 - A \\ d_{22}c_2 - A \\ \vdots \\ d_{nn}c_n - A \end{bmatrix}.$$

- Thus, the SPNE is the equilibrium where each firm $i \in \mathcal{N}$ plays p_i^{NE} in each subgame.

Outline

- Let $\kappa \subseteq N$ denote the set of cartel members.
- Cartel members set prices to maximize their *joint* profit $\Pi^C = \sum_{j \in \kappa} (p_j - c_j) Q_j$
- FOCs for each $j \in \kappa$ in the general case (FOCs for non-members are the same as in the NE):

$$Q_j(p_j, p_{-j}) + \sum_{i \in \kappa} \frac{\partial Q_i(p_i, p_{-i})}{\partial p_j}(p_i - c_i) = 0 \qquad \text{, for } i, j \in \kappa$$

- With linear demand, the FOCs for each $j \in \kappa$ are:

$$\sum_{i \in \kappa} (d_{ij} + d_{ji}) p_i + \sum_{l \notin \kappa, l \in \mathcal{N}} d_{jl} p_l = \sum_{i \in \kappa} d_{ij} c_i - A$$

Equilibrium prices with collusion II

- Combining FOCs for cartel members and non-members with linear demand, we can find the analytical solution for the equilibrium prices (for both colluding and non-colluding firms):

$$\mathbf{p}^* = (\mathbf{D}^*)^{-1}\mathbf{B}^* \qquad \text{, where}$$

$$\mathbf{D}^* = \begin{bmatrix} D_{11} & \dots & D_{1n} \\ \vdots & \ddots & \vdots \\ D_{n1} & \dots & D_{nn} \end{bmatrix} \qquad , D_{ij} = \begin{cases} (d_{ij} + d_{ji}) & \text{if } i, j \in \kappa \\ d_{ij} & \text{otherwise} \end{cases}$$

and

$$\mathbf{B}^* = \begin{bmatrix} B_1 \\ B_2 \\ \vdots \\ B_n \end{bmatrix} \qquad , B_i = \begin{cases} \left(\sum_{m \in \kappa} d_{mi} c_m \right) - A & \text{if } i \in \kappa \\ d_{ii} c_i - A & \text{otherwise} \end{cases} .$$

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Incentive compatibility and cartel break-down

- For a cartel to exist in equilibrium, deviation from the collusive agreement and subsequent deviation to the Nash equilibrium must be (weakly) less lucrative than continuing in the collusive agreement for all cartel members $i \in \kappa$.

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- Let:
  - δ denote the firms' common discount factor
  - $\Pi_i^C = (p_i^* - c_i)Q_i(p_i^*, \mathbf{p}_{-i}^*)$ denote firm i's profits when it part-takes in collusion (C)
  - $\Pi_i^D = (p_i^D - c_i)Q_i(p_i^D, \mathbf{p}_{-i}^*)$ denote firm i's profits when it deviates from the collusive agreement (D)
  - $\Pi_i^{NE} = (p_i^{NE} - c_i)Q_i(p_i^{NE}$ denote firm i's profits when it plays the Nash equilibrium (NE)
- The incentive compatibility constraint (ICC) for $i \in \kappa$ is given by:

$$\underbrace{\frac{1}{1-\delta}\Pi_i^C}_{\text{NPV of C}} \geq \underbrace{\Pi_i^D}_{\text{One-period profit from D}} + \underbrace{\frac{\delta}{1-\delta}\Pi_i^{NE}}_{\text{NPV of NE for all subsequent periods}}$$

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- If each cartel member is sufficiently patient, namely if

$$\delta \geq \frac{\Pi_i^D - \Pi_i^C}{\Pi_i^D - \Pi_i^{NE}} \equiv \delta_{min}$$

for all $i \in \kappa$, then the cartel can indeed be an equilibrium of the repeated price game.

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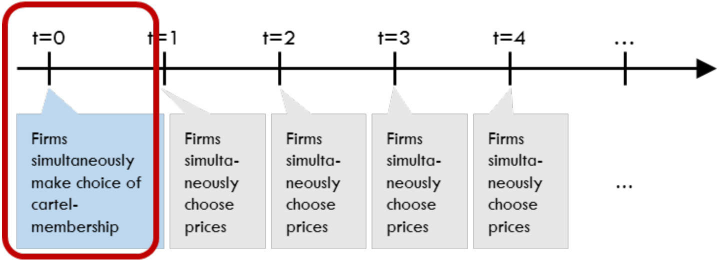

Outline

Cartel stability – Symmetric firms I

- Symmetry between firms implies that $d_{ii} = d_{jj} \equiv d$ and $d_{ij} = d_{ji} = d_{ik}$ for all $i, j, k \in \mathcal{N}$.
- I have therefore reduced the problem of N FOCs and N unknowns to only two types of FOCs – one for cartel members and one for non-members.

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- Let η denote the share of sales that exit the market following a price increase, such that:

$$(1-\eta)d = -\sum_{i=1}^{N-1} d_{ji} = -(N-1)d_{ij} \qquad \Leftrightarrow \qquad d_{ij} = \frac{-d(1-\eta)}{(N-1)}$$

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- Furthermore, let N denote the number of firms in the industry, and K the number of cartel-members. Profit functions are thus given by:

$$\Pi^C = \sum_{j \in \kappa} (p_j - c) \left(A + dp_j - \tfrac{(N-K)}{(N-1)}(1-\eta)dp_{NM} + \tfrac{-d(1-\eta)}{(N-1)} \sum_{l \in \kappa, l \neq j} p_l \right)$$

$$\Pi_i = (p_i - c) \left(A + dp_i - \tfrac{(N-K-1)}{(N-1)}(1-\eta)dp_{NM} - \tfrac{(K)}{(N-1)}(1-\eta)dp_\kappa \right) \qquad , \text{ for } i \notin \kappa$$

Cartel stability – Symmetric firms II

The solution to the repeated price game for a cartel of size K is thus given by the following prices and profits:

- $p_{NM}(K) = \frac{(N-1)\left[c+\frac{A}{-d}\right]\left[2(N-1)-(K-2)(1-\eta)\right]+K(K-1)(1-\eta)^2 c}{2\left[2(N-1)-(N-K-1)(1-\eta)\right]\left[(N-1)-(K-1)(1-\eta)\right]-\left[K(N-K)(1-\eta)^2\right]}$

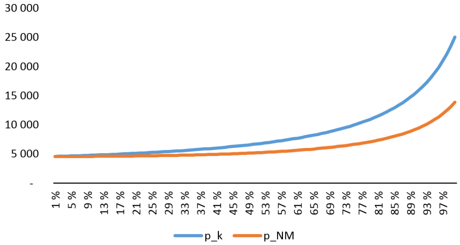
- $p_{\kappa}(K) = \frac{(N-1)\left[c+\frac{A}{-d}\right]-c(K-1)(1-\eta)}{2((N-1)-(K-1)(1-\eta))} + \frac{(N-K)(1-\eta)}{2((N-1)-(K-1)(1-\eta))} p_{NM}$

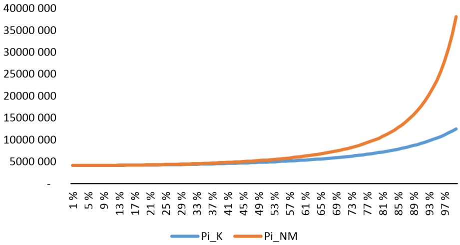
- $\Pi_{\kappa}(K) = (p_{\kappa}(K) - c)\left(A + \frac{(N-K+\eta(K-1))}{(N-1)} dp_{\kappa}(K) - \frac{(N-K)}{(N-1)}(1-\eta)dp_{NM}(K)\right)$

- $\Pi_{NM}(K) = (p_{NM}(K) - c)\left(A + \frac{K+\eta(N-K-1)}{(N-1)} dp_{NM}(K) - \frac{K}{(N-1)}(1-\eta)dp_{\kappa}(K)\right)$

Cartel stability – Symmetric firms III

- Prices and profits for cartel members and non-members by degree of cartelization $\frac{K}{N}$ for parameter values: $A = 1000$, $N = 100$, $d = -0.2$, $c = 10$, and $\eta = 0.1$:





Internal and external cartel stability

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Definition: Cartel stability

A cartel in a market with symmetric firms, linear demand and differentiated price competition, such that $d_{ii} = d_{jj} \equiv d$ and $d_{ij} = d_{ji} = d_{ik}$ for all $i, j, k \in \mathcal{N}$, is:

- **Internally stable** iff $\Pi_{\kappa}(K) \geq \Pi_{NM}(K-1)$
- **Externally stable** iff $\Pi_{NM}(K) \geq \Pi_{\kappa}(K+1)$

- The concept was first introduced by d'Aspremont et al. (1983).
- A cartel is considered *internally* stable if no participant prefers to leave the cartel.
- A cartel is considered *externally* stable if no free-rider prefers to join the cartel.

Definition: Cartel stability

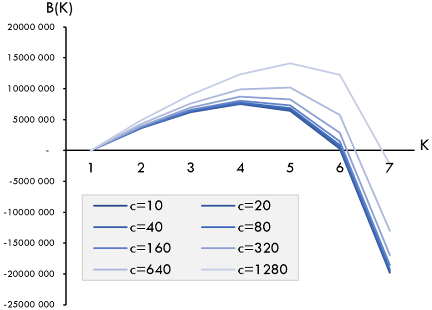
A cartel in a market with symmetric firms, linear demand and differentiated price competition, such that $d_{ii} = d_{jj} \equiv d$ and $d_{ij} = d_{ji} = d_{ik}$ for all $i, j, k \in \mathcal{N}$, is:

- **Internally stable** iff $\Pi_{\kappa}(K) \geq \Pi_{NM}(K-1)$
- **Externally stable** iff $\Pi_{NM}(K) \geq \Pi_{\kappa}(K+1)$

- Define the net benefit of collusion as $B(K) = \Pi_{\kappa}(K) - \Pi_{NM}(K-1)$
- The optimal K is thus the maximum K such that $B(K) \geq 0$
  - The shape of $B(K)$ and the optimal K depend on the parameter values: N, c, A, d and η

Benefit of collusion $B(K)$ – effect of parameter values I

- Changes in marginal cost c or price sensitivity d have little impact on the optimal cartel size:



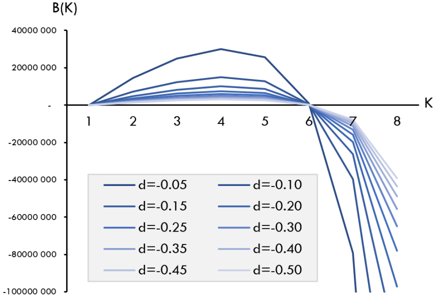


Figure: **Net benefit from collusion ($B(K)$)** – sensitivities for parameters c and d. Standard parameter values: $A = 1000$, $N = 8$, $d = -0.2$, $c = 10$, and $\eta = 0.1$

Benefit of collusion $B(K)$ – effect of parameter values II

- When exit η becomes small, mulitple equilibria may exist:
  - A market-wide cartel faces the demand
    $$Q^C = A + dp_K - \underbrace{\frac{d(1-\eta)}{N-1}}_{d_{ij}}(N-1)p_K = A + \eta dp_K.$$
  - When $\eta \to 0$, demand becomes independent of price. This causes $B(K)$ to jump up for full cartelization.

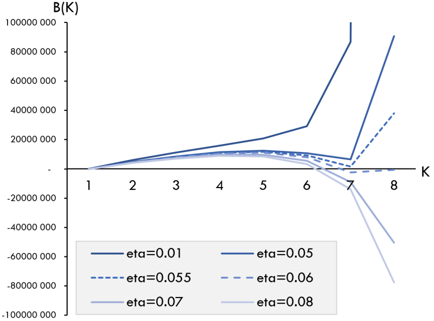


Figure: **Net benefit from collusion ($B(K)$)** – sensitivities for η. Standard parameter values: $A = 1000$, $N = 8$, $d = -0.2$, $c = 10$, and $\eta = 0.1$

Benefit of collusion $B(K)$ – effect of parameter values III

- Changes in marginal industry size N alters the shape of $B(K)$. In small industries, full cartelization is optimal.
  - Du to the definition $(1-\eta)d = -(N-1)d_{ij}$, a small industry implies large cross-derivatives → the cartel internalizes a larger share of the externality per extra cartel member.
- Full cartelization is only optimal for concentrated industries.
- Due to the mechanical relationship between N and d_{ij} it may be closeness in competition, rather than industry size that determines cartel size.

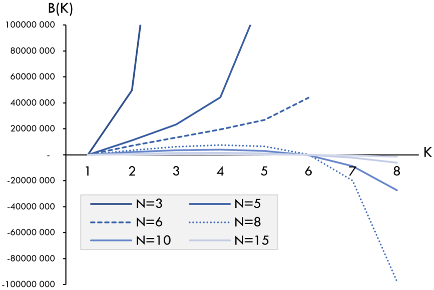


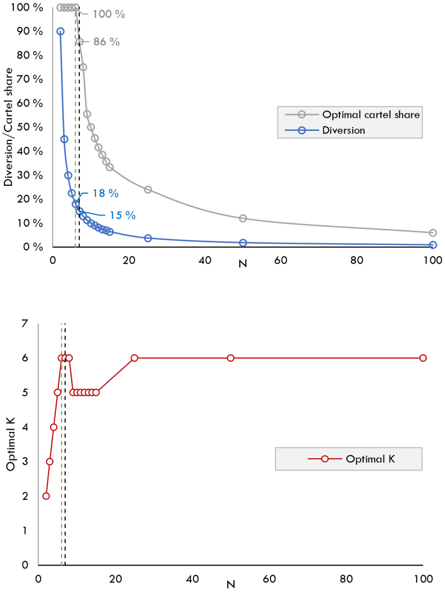
Figure: **Net benefit from collusion ($B(K)$)** – sensitivities for N. Standard parameter values: $A = 1000$, $N = 8$, $d = -0.2$, $c = 10$, and $\eta = 0.1$

Closeness in competition and cartel size I

- Let the diversion ratio from firm i to firm j be defined as

$$D_{ij} \equiv \frac{\frac{\partial Q_j}{\partial p_i}}{\frac{\partial Q_i}{\partial p_i}} = -\frac{d_{ji}}{d}$$

- The figures show diversion ratios, optimal cartel shares and sizes of stable cartels (K) for different industry sizes N with symmetric product differentiation. (Parameter values: $A = 1000$, $d = -0.2$, $c = 10$, and $\eta = 0.1$)
- Illustrates relationship between optimal cartel *share* and diversion.
- Illustrates that cartel *size* remains relatively stable at 5-6 cartel members.



- Data on 33 US cartels in the period 1963-1972
- Theoretical values for one specific set of parameter values
- Suggestive of a cartel share that is declining in industry size.

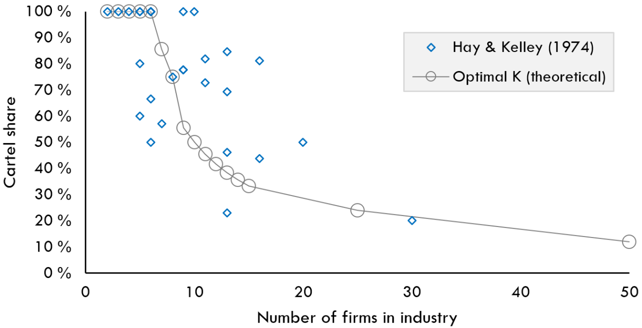


Figure: Distributions of cartel share and industry size, based on data from [?] and theoretical model with parameter values: $A = 1000$, $d = -0.2$, $c = 10$, and $\eta = 0.1$

Outline

Incentive compatibility of stable cartels

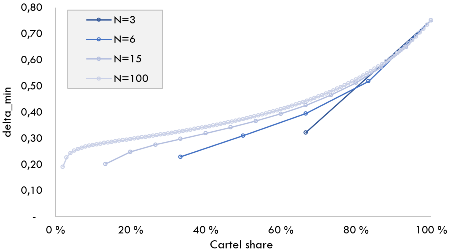
- So far, I have assessed cartel stability. It remains to show whether a stable cartel is incentive compatible.
- A cartel of size K is incentive compatible iff firms are sufficiently patient:

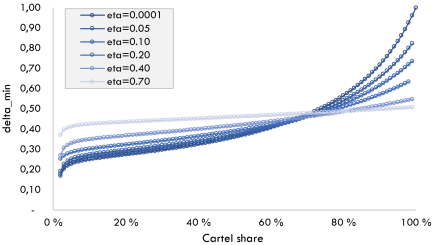
$$\delta \geq \delta_{min}(K) \equiv \frac{\Pi^D(K) - \Pi_\kappa(K)}{\Pi^D(K) - \Pi_\kappa(1)},$$

  where Π^D is the profit a cartel member earns when deviating and using that $\Pi_\kappa(1) = \Pi_{NM}(1)$.
- Note that $\delta_{min} \leq 1$, as $\Pi_\kappa(K) \geq \Pi_\kappa(1)$ for all $K \leq N$. Thus, a stable cartel is always incentive compatible if firms are sufficiently patient.
- The question remains: Exactly how patient do firms have to be?

Incentive compatibility of stable cartels I

- The figures show values for δ_{min} – sensitivities for parameters N and η. Standard parameter values: $A = 1000$, $d = -0.2$, $c = 10$, $N = 100$ and $\eta = 0.1$
- Numerical results suggest that $\frac{\partial \delta_{min}(K)}{\partial K} \geq 0$, such that δ_{min} is largest with full cartelization.
- If η is sufficiently large, δ_{min} remains relatively low. $\rightarrow$ ICC likely to hold, even with full cartelization.
- Consequently, the conditions for cartel stability will determine optimal cartel size.





Summary

Research questions:

(i) Under what industry conditions can we expect a cartel to be industry-wide?
(ii) What market characteristics matter the most for the degree of cartelization?
(iii) Which firms are most likely to form a cartel?

Lessons from the model

(i) When products are differentiated, cartelization will be larger in industries where products are closer substitutes.
(ii) Cartelization will also be larger in more concentrated markets.
(iii) The optimal cartel size remains relatively constant at 5-6 firms, regardless of industry size.
(iv) The larger the degree of cartelization, the more damaging the cartel. This effect is increasing in the cartel share.

Extensions and further research

The road ahead:

- Comparative statics: showing mathematically how $B(K)$ and δ_{min} change with K. (Current model)

[1] An advantage of not assuming full cartelization is that there is no need to define a market (and which firms should be included) in order to determine coordinated effects . This would correspond well with current models for measuring unilateral effects.

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Possible extensions:

- Including competition policy in model, e.g. through risk of detection.
- Introduce other membership rules for the first stage of the game.
- Allow for multiple coalitions (trivial for the symmetric case)

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