Coordinated effects

– An index

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Outline

Why an index?

- When analyzing mergers, competition authorities (CAs) assess:
  - Unilateral effects
  - Coordinated effects

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- There are several indices that allow CAs to quantify unilateral effects. See for example:
  - Farrell and Shapiro (2010)
  - Hausman, Moresi, and Rainey (2011)
  - Moresi and Salop (2013)
  - Asphjell et al. (2017)

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- Currently no index for coordinated effects:
  - Without quantification, difficult for CAs to prove substantial coordinated effects (burden of proof).
  - Consequently, mergers are rarely stopped on the grounds of coordinated effects.
  - Empirical studies have estimated coordinated effects due to mergers, for example in the US brewing industry.

How to measure incentives for collusion?

- EC Horizontal Merger Guidelines (§39): "*A merger in a concentrated market may significantly impede effective competition, through the creation or the strengthening of a collective dominant position, because it increases the likelihood that firms are able to coordinate their behaviour in this way and raise prices even without entering into an agreement...within the meaning of Art. 81*".

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- How to measure the likelihood of coordination?
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- How to measure the likelihood of coordination?
  - ⇒ If firms make choices based payoffs, the relative magnitude of collusive and deviation payoffs hold information on the probability of collusion.

- How to measure *changes* in the probability of collusion?
  - ⇒ A merger will affect collusive and deviation payoffs – and hence change the probability of collusion.

Outline

Two index options

- The discrete time incentive compatibility constraint for firm i is:

$$\Pi_{i,t}^{C} + \delta V_{i,t}^{C} \geq \Pi_{i,t}^{D} + \delta_i V_{i,t}^{D}$$

Two index options

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- Using grim-trigger strategies, this can be rewritten as:

$$\frac{1}{1-\delta_i}\Pi_{i,t}^{C} \geq \Pi_{i,t}^{D} + \frac{\delta_i}{1-\delta_i}\Pi_{i,t}^{NE}, \tag{1}$$

where $t = \{pre, post\}$ denotes time before/after the merger.

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Alternative 1: Incentive compatibility constraint ratio (ICCR) pre/post merger

$$ICCR_{i,t} \equiv \frac{\Pi^C_{i,t}}{(1-\delta_i)\Pi^D_{i,t} + \Pi^{NE}_{i,t}} \tag{2}$$

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Alternative 2: Critical patience (δ_{min}) pre/post merger

$$\delta_{min,i,t} \equiv \frac{\Pi^D_{i,t} - \Pi^C_{i,t}}{\Pi^C_{i,t} - \Pi^{NE}_{i,t}} \tag{3}$$

Outline

Important assumptions

- Same assumptions as indices for unilateral effects:
  - Products are differentiated.
  - Firms set prices simultaneously each period.
  - Demand functions are linear in prices.
  - Firms produce at constant marginal cost (no efficiency gains).

- Additional assumptions:
  - Complete internalization of competition between colluding firms.
  - Grim-trigger punishment strategies

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  - Products are differentiated.
  - Firms set prices simultaneously each period.
  - Demand functions are linear in prices.
  - Firms produce at constant marginal cost (no efficiency gains).
- Additional assumptions:
  - Complete internalization of competition between colluding firms.
  - Grim-trigger punishment strategies
- Observable entities are mostly the same as the ones needed in indices for unilateral effects:
  - Pre-merger prices
  - Pre-merger marginal costs
  - Pre-merger quantities
  - Diversion ratios
  - Firms' patience (not needed for unilateral effects)

  Main difference compared to unilateral effect: Entities are needed for all firms in the market.

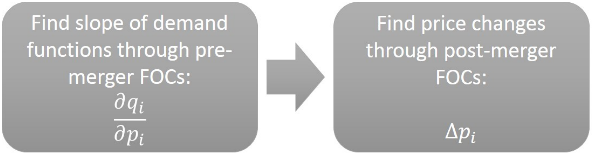
Outline

Finding counterfactual profits – *illustration with single product firms*

Find slope of demand functions through pre-merger FOCs:

$$\frac{\partial q_i}{\partial p_i}$$

Finding counterfactual profits – *illustration with single product firms*



Finding counterfactual profits – *illustration with single product firms*

Find slope of demand functions through pre-merger FOCs:

$$\frac{\partial q_i}{\partial p_i}$$

→

Find price changes through post-merger FOCs:

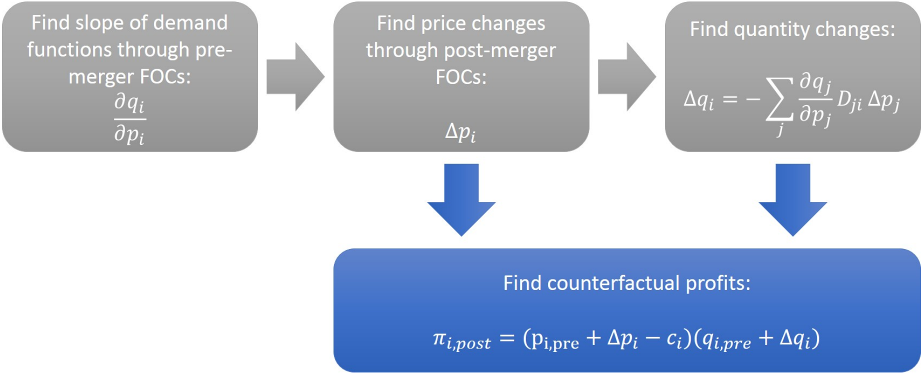
$$\Delta p_i$$

→

Find quantity changes:

$$\Delta q_i = -\sum_j \frac{\partial q_j}{\partial p_j} D_{ji}\, \Delta p_j$$

Finding counterfactual profits – *illustration with single product firms*



Pre-merger first order conditions

- Each firm maximizes the joint profit of its products $i \in I_t$:

$$\max_{\{p_i\}_{i \in I_t}} \Pi_{I,t} = \sum_{i \in I_t} \Pi_{i,t} = \sum_{i \in I_t} (p_{i,t} - c_i) q_{i,t}$$

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$$q_{i,pre} + \sum_{j \in I_{pre}} (p_{j,pre} - c_j) \frac{\partial q_j}{\partial p_i} = 0$$

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- Thus, the FOC gives us the slope of the demand function:

$$\frac{\partial q_i}{\partial p_i} = \frac{q_{i,pre}}{\sum_{j \in I_{pre}} (p_{j,pre} - c_j) D_{ij}} \qquad \text{, where } D_{ij} \equiv -\frac{\frac{\partial q_j}{\partial p_i}}{\frac{\partial q_i}{\partial p_i}} \tag{4}$$

- As demand is linear, the change in quantity can be calculated as:

$$\Delta q_i = q_{i,post} - q_{i,pre} = -\sum_j \frac{\partial q_j}{\partial p_j} D_{ji} \Delta p_j \tag{5}$$

Post-merger first order conditions

- The post-merger FOC for product *i* is similar to the pre-merger FOC:

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- Substituting for $q_{i,post}$ from (5), and using that $p_{i,post} = p_{i,pre} + \Delta p_i^{NE}$:

$$\sum_{j \in N} \frac{\frac{\partial q_j}{\partial p_j}}{\frac{\partial q_i}{\partial p_i}} D_{ji} \Delta p_j^{NE} + \sum_{j \in I_{post}} D_{ij} \Delta p_j^{NE} = \sum_{j \in I_{pre}} (p_{j,pre}^{NE} - c_j) D_{ij} - \sum_{l \in I_{post}} (p_{j,pre}^{NE} - c_j) D_{ij}$$

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- The FOCs can be rewritten in matrix notation:

$$(\mathbf{A} + \Omega_{post}^{NE} \circ \mathbf{D}) \Delta \mathbf{p}^{NE} = \left((\Omega_{pre}^{NE} - \Omega_{post}^{NE}) \circ \mathbf{D} \right) \mathbf{M} \qquad (6)$$

where:

- element a_{ij} of matrix $\mathbf{A}$ is equal to $\frac{\frac{\partial q_j}{\partial p_j}}{\frac{\partial q_i}{\partial p_i}} D_{ji}$, where $i \in I_{pre}$ and $j \in J_{pre}$.
- Ω_t^{NE} is the ownership matrix at time $t = \{pre, post\}$, where element ω_{ij} is equal to 1 if products i and j are owned by the same firm and 0 otherwise.
- element d_{ij} of matrix $\mathbf{D}$ is equal to the diversion from product i to j: D_{ij}.
- $\Delta \mathbf{p}^{NE}$ is the vector of price changes.
- $\mathbf{M}$ is the vector of margins, where the i-th element m_i is equal to $(p_{i,pre}^{NE} - c_i)$.

Price changes and counterfactual profit

- Equation (6) can be solved for the vector of price changes $\Delta\mathbf{p}^{NE}$
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Price changes and counterfactual profit

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- Note that $\Delta \mathbf{p}^{NE}$ are the unilateral effects of the merger.
- We can find the changes in quantities from equation (5): $\Delta \mathbf{q}^{NE}$
- The firms' new profit is:

$$\Pi_{I,post}^{NE} = \sum_{i \in I_{post}} \Pi_{i,post}^{NE} = \sum_{i \in I_{post}} (p_{i,pre}^{NE} + \Delta p_i^{NE} - c_i)(q_{i,pre}^{NE} + \Delta q_i^{NE}) \tag{7}$$

Summary of results – coordination ant Nash equilibium

- Vectors of price changes for the new Nash equilibrium (unilateral effects) and coordination differ only in ownership matrices:

$$\Delta \mathbf{p}^{NE} = \left(\mathbf{A} + \Omega^{NE}_{post} \circ \mathbf{D}\right)^{-1} \left((\Omega^{NE}_{pre} - \Omega^{NE}_{post}) \circ \mathbf{D}\right) \mathbf{M} \quad (8)$$

$$\Delta \mathbf{p}^{C}_{pre} = \left(\mathbf{A} + \Omega^{C}_{pre} \circ \mathbf{D}\right)^{-1} \left((\Omega^{NE}_{pre} - \Omega^{C}_{pre}) \circ \mathbf{D}\right) \mathbf{M} \quad (9)$$

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- We can therefore find the counterfactual profits:

$$\Pi^{NE}_{I,pre} = \sum_{i \in I_{pre}} \Pi^{NE}_{i,pre} = \sum_{i \in I_{pre}} (p^{NE}_{i,pre} - c_i)(q^{NE}_{i,pre})$$

$$\Pi^{NE}_{I,post} = \sum_{i \in I_{post}} \Pi^{NE}_{i,post} = \sum_{i \in I_{post}} (p^{NE}_{i,pre} + \Delta p^{NE}_{i} - c_i)(q^{NE}_{i,pre} + \Delta q^{NE}_{i})$$

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Deviation profits (I)

- Simplest to calculate price changes relative to collusive prices.
- The deviating firm maximizes the joint profit of its own products. $\Rightarrow$ FOCs similar to previous expressions.
- As we are comparing to the *coordination* outcome, $\Delta p_j^D = 0$ and $\Delta q_j^D = 0$ for all $j \notin I_t$.

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- The FOCs for products $i \in I_t$ are:

$$\sum_{j \in I_t} \left(\frac{q_{j,pre}^{NE}}{q_{i,pre}^{NE}} D_{ji} + D_{ij} \right) \Delta p_{j,t}^D = \frac{q_{i,t}^C}{q_{i,pre}^{NE}} \sum_{j \in I_{pre}} (p_{j,pre}^{NE} - c_j) D_{ij} - \sum_{j \in I_t} (p_{j,pre}^{NE} - c_j) D_{ij}$$

Deviation profits (II)

- The price changes can be written in matrix form:[1]

$$\Delta \mathbf{p}^D_{l,t} = \mathbf{B}^{-1}\mathbf{C}_t \tag{11}$$

where:

- $\Delta \mathbf{p}^D_{l,t}$ is the vector of price changes relative to the coordination price for firm l at time t.
- element b_{ij} in matrix $\mathbf{B}_t$ is equal to $\frac{q^{NE}_{j,pre}}{q^{NE}_{i,pre}} D_{ji} + D_{ij}$
- the i-th element of the vector $\mathbf{C}_t$ is equal to $\frac{q^C_{i,t}}{q^{NE}_{i,pre}} \sum_{j \in I_{pre}} (p^{NE}_{j,pre} - c_j) D_{ij} - \sum_{j \in I_t} (p^{NE}_{j,pre} - c_j) D_{ij}$

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- We can therefore find deviation profits:

$$\Pi^D_{l,t} = \sum_{i \in I_t} \Pi^D_{i,t} \quad = \sum_{i \in I_t} (p^{NE}_{i,pre} + \Delta p^C_{i,t} + \Delta p^D_{i,t} - c_i)(q^{NE}_{i,pre} + \Delta q^C_{i,t} + \Delta q^D_{i,t}) \quad (12)$$

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Outline

Choice of ownership matrices

- Recall that:[2]

$$\Delta \mathbf{p}^{NE} = \left(\mathbf{A} + \Omega^{NE}_{post} \circ \mathbf{D}\right)^{-1} \left(\left(\Omega^{NE}_{pre} - \Omega^{NE}_{post}\right) \circ \mathbf{D}\right) \mathbf{M}$$

$$\Delta \mathbf{p}^{C}_{pre} = \left(\mathbf{A} + \Omega^{C}_{pre} \circ \mathbf{D}\right)^{-1} \left(\left(\Omega^{NE}_{pre} - \Omega^{C}_{pre}\right) \circ \mathbf{D}\right) \mathbf{M}$$

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$$\Delta \mathbf{p}^{D}_{l,t} = \mathbf{B}^{-1} \mathbf{C}_t$$

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$$\Delta \mathbf{p}^{C}_{post} = \left(\mathbf{A} + \Omega^{C}_{post} \circ \mathbf{D}\right)^{-1} \left((\Omega^{NE}_{pre} - \Omega^{C}_{post}) \circ \mathbf{D}\right) \mathbf{M}$$
$$\Delta \mathbf{p}^{D}_{l,t} = \mathbf{B}^{-1}\mathbf{C}_t$$

- Ω^{NE}_{post} and Ω^{NE}_{pre} are given by the pre-/post-merger market structure.

- Ω^{C}_{pre} and Ω^{C}_{post} are *choices*. Two important candidates:
  - The all-inclusive cartel
  - Two-firm cartels

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Choice of ownership matrices

- The all-inclusive cartel:
  - Note that all elements in Ω_t^C are equal to 1, s.t. $\Omega_{pre}^C = \Omega_{post}^C$.
  - Thus, $\Delta \mathbf{p}_{pre}^C = \Delta \mathbf{p}_{post}^C$.
  - It follows that $\Pi_{I,pre}^C = \Pi_{I,post}^C$.

$\Rightarrow$ *Coordinated effects (changes in ICCs) likely biggest for the merging parties.*

[3] Note that $\Omega_{\{IJ,K\}}^C = \Omega_{\{IJK\}}^{NE}$.

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- Two-firm cartels:
  - Let I, J (where $I \neq J$) denote the merging firms, and let $K(\neq I \neq J)$ denote an outside firm.
  - Relevant pre-merger (two-firm) cartels including these firms are $\{I, K\}$, $\{J, K\}$. Let the corresponding ownership matrices be denoted by $\Omega_{\{I,K\}}^C$ og $\Omega_{\{J,K\}}^C$.
  - Post-merger, the only two-firm cartel including these firms is $\{IJ, K\}$, where IJ denotes the merged firm.[3]

  $\Rightarrow$ *Coordinated effects likely biggest for the outside firm.*

[3] Note that $\Omega_{\{IJ,K\}}^C = \Omega_{\{IJK\}}^{NE}$.

Outline

Work remaining

- Choice of index (Alternative 1 or 2)
- Stylized examples of markets to test the effect of a maverick firm on coordinated effects
- Stylized examples of markets to test the effect of market definition on coordinated effects
- Include discussion on measurement of the firms' patience δ_i
- Possibly test the index on data from the telecom market in Norway
- Possibly include efficiency gains