Dynamic Duopoly with Collusion and Horizontal Product Differentiation

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Abstract

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In this paper, I analyze the effect of collusion on investments in horizontal product differentiation. In the benchmark static model, I find that if firms are sufficiently patient, they invest less in product differentiation if the industry is cartelized. Intuitively, firms that compete will be more willing to invest in product differentiation if they are patient, as they are more willing to pay for an investment today to soften long-run price competition. For a cartelized industry, patient firms imply that collusion is easier to sustain. As firms must invest to disincentivize deviation (and lower the critical discount factor), high patience implies that collusion can be sustained at lower investment levels. Thus, I find that partial collusion fosters less investment (and less product differentiation) if firms are sufficiently patient. However, if firms are sufficiently impatient, the result is reversed. In the continuation of this project, I aim to compare the static model to a fully dynamic investment model.

1 Introduction and research questions

It is well known that cartels are harmful to consumers, as they raise prices and limit production. When firms collude on price or quantity, competition along other dimensions may also be affected. For example, cartel members may make different investment choices than firms that compete. If the collusive agreement leads to tougher competition on product innovation and quality, the effect of a cartel on consumer surplus is ambiguous. On one hand, collusion leads to higher prices. On the other hand,

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it may lead to improvements in quality or product variety that increase consumers' utility.

In reality, markets rarely consist of perfect substitutes, but rather of a limited number of differentiated firms or product varieties. Furthermore, consumers are likely to prefer one product variety to another, and may differ in their preferences. This is commonly referred to as horizontal product differentiation. Incentives for collusion, cartel stability and cartel formation may differ in markets with homogeneous and differentiated products. Conversely, firms' incentives to invest in product innovation, affecting how differentiated their products are, may change if a market i cartelized. This implies that product differentiation is not an exogenous parameter, but in fact an endogenous choice.

Relevant research questions may therefore be:
i) How does collusion affect incentives to invest in product innovation?
ii) When firms make dynamic investment decisions, how does collusion affect efficiency?

In this research project, I plan to compare results of the dynamic investment model to the benchmark case of static investment. In the following, I will first give a brief review of the articles most closely related to this project. Then, I will present the benchmark model of static investment, followed by a brief outline of the dynamic model. I also present preliminary results for the benchmark case and expected types of results for the dynamic model. Finally, I summarize work remaining.

2 Brief literature review

The question of how collusion affects investment incentives is not new. Common in this literature is the assumption that firms may collude on prices or quantities, but not on investment decisions. See for example Fershtman & Gandal (1994), Davidson & Deneckere (1990), or Fershtman & Gandal (1994). There are strong arguments for why collusion on investment choices is difficult. Investment is difficult to monitor and (e.g. R&D) output is uncertain. Furthermore, empirical observations of cartels support the assumption. As firms only collude on one dimension (e.g. price), but not on other (e.g. investment), this is often referred to as semi-collusion. As in my benchmark model, investment is modeled as a one-shot choice at the first stage of the game. The initial investment stage is followed by an infinitely repeated price game, where firms may collude.

In the investment stage in Davidson & Deneckere (1990), firms choose investment in capacity. The authors analyze how collusion affects the firms' investments in (excess) capacity. Fershtman & Gandal (1994) show that collusion may lower overall profits, as it intensifies competition on other dimensions than price. They also show that firms suffer from a commitment problem, as they could earn higher profits by committing not to collude. In my baseline model, I similarly find that firms may prefer not to collude if they are sufficiently impatient. Collusion requires firms to make large and costly investments in product innovation (to increase product differentiation), to lower the gains from deviation. If firms invest less, collusion is no longer incentive compatible and firms do not suffer from a commitment problem.

The model of Friedman & Thisse (1993) assumes that firms compete in a spacial product differentiation model, the co-called Hotelling model (1929). In their duopoly model, firms first choose their location on a line segment. Subsequently, they engage in an infinitely repeated price game. They can collude in the price game, but not in their investment decision. In this regard, their model closely resembles my benchmark model. Instead of a spacial product differentiation model, I use a representative consumer approach. The formulation of the utility function I use dates back to Levitan & Shubik (1971), and is used in several textbooks, such as Vives (2001) and Motta (2004). The main result of Friedman & Thisse (1993) is that "*Partial collusion fosters minimum product differentiation*", as stated in the title of their paper. Similar to their findings, I find that partial collusion fosters less investment (and less product differentiation) if firms are sufficiently patient. However, if firms are sufficiently impatient, the result is reversed.

It is unrealistic to assume that investment is a one-shot decision. Furthermore, it seems unlikely that firms choose their investments while anticipating that collusion will occur in the future. It is more reasonable to assume that investments are repeated, dynamic decisions. Furthermore, investment – particularly in R&D activities yields uncertain results. This uncertainty should be included in a model. Ericson & Pakes (1995) build a comprehensive numerical model with dynamic investment, horizontal product differentiation and firm entry/exit. Fershtman & Pakes (2000) extend the model to allow for price collusion. My dynamic model will rely heavily on the latter.

Fershtman & Pakes (2000) find that collusion increases product quality and varieties, and decreases concentration. If products instead were horizontally differentiated, one firm's investment in product R&D benefits its opponents instead of harming him. My dynamic model, differs from that of Fershtman & Pakes (2000), as I assume horizontal (instead of vertical) product differentiation. Unlike their model, aggregate demand is not constant and depends on the firms' prices. Furthermore, I wish to isolate the effect of collusion of product differentiation by fixing the number of firms to $N = 2$. Finally, I want to compare the effect of collusion on investment in both the benchmark (static) model and the dynamic model.

3 The model

3.1 Benchmark: Static investment decision

Figure 1 illustrates the timing of the static investment game. In the first period, firms simultaneously choose their level of investment. Firms only make the investment choice once. Subsequently, firms play an infinitely repeated price-game with differentiated products. In each stage-game of the price-game, firms simultaneously choose prices. I will assume that cartel break-down will lead to a reversion to the Nash equilibrium of the stage game for all future periods (the punishment strategy is a so-called trigger strategy).

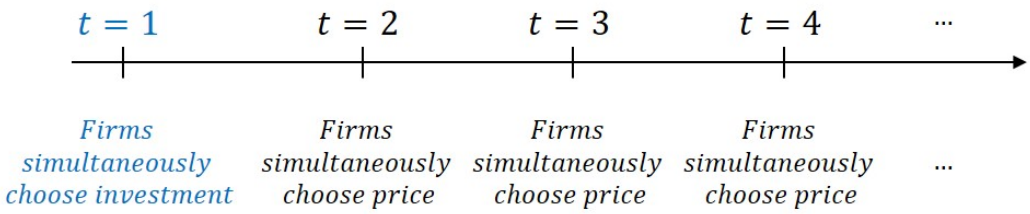

Figure 1: Timing of the game

3.1.1 Modelling consumers

I will assume that the demand side of the industry can be modeled by a representative consumer with quadratic utility function:

$$U(\mathbf{q}) = v \sum_{i=1}^{N} q_i - \frac{N}{2(1+\mu)} \left[\sum_{i=1}^{N} q_i^2 + \frac{\mu}{N} \left(\sum_{i=1}^{N} q_i \right)^2 \right] \tag{1}$$

where I is the numeraire good, N is the number of firms/products and $\mu \in [0, \infty)$ represents the level of symmetric product differentiation. This formulation of the utility function dates back to Levitan & Shubik (1971), and is used in several textbooks, such as Vives (2001) and Motta (2004). Marshallian demand for product i is:

$$q_i = \frac{1}{N} \left[v - p_i - \mu \left(p_i - \frac{1}{N} \sum_{j=1}^{N} p_j \right) \right] \tag{2}$$

Demand for product i directly decreases in firm i's own price. Additionally, demand is indirectly affected by how much firm i's price differs from the average price in the industry. If p_i is below the average market price, demand for firm i's product increases. Conversely, if p_i is above the average market price, demand for firm i's product decreases. How large this indirect effect is, depends on the level of product differentiation. The more differentiated the products are (the smaller μ), the smaller is the indirect effect. Another attractive feature of this formulation of the demand functions, as Motta (2004) points out, is that aggregate demand is independent of the substitutability between the products.

For a duopoly, Marshallian for product i is:

$$q_i(p_i, p_j) = \frac{1}{2}\left[v - \frac{(2+\mu)}{2}p_i + \frac{\mu}{2}p_j\right] \tag{3}$$

3.1.2 Modelling firms

Assume that costs are linear. If firms do not collude, they set prices to maximize their own profit. Thus, for each $i = 1, 2$ the optimization problem is:

$$\max_{p_i}(p_i - c)q_i(p_i, p_j) \tag{4}$$

Assume that, under collusion, firms set prices to maximize the cartel's joint profit. Thus, the collusive maximization problem is:

$$\max_{\{p_i\}_i} \sum_j (p_j - c)q_j(p_j, p_{-j}) \tag{5}$$

For simplicity, assume that firms have symmetric marginal costs that are equal to zero. Namely, $c_i = 0$ for $i = 1, 2$.

3.1.3 Investment in product innovation

Assume that it is costly for firms to invest in product innovation and differentiation. Furthermore, assume that the achieved level of product differentiation μ is a function of the two firms' investment choices x_i for $i = 1, 2$:

$$\mu = M(x_i, x_j), \tag{6}$$

where M is decreasing and convex in both arguments. The cost of investment for firm i hence depends on the opponents investment level and the desired level of product differentiation μ. The assumptions on the curvature and slope of M imply that investment costs are convex in product differentiation. Intuitively, this assumption seems plausible. As a firm starts to invest in product innovation and branding, it becomes increasingly harder to invent new and different characteristics.

Furthermore, assume the following asymptotic properties:

$$\lim_{x_i \to 0} \lim_{x_j \to 0} M(x_i, x_j) = \infty \tag{7}$$

$$\lim_{x_i \to 0} \lim_{x_j \to 0} \frac{\partial M(x_i, x_j)}{\partial x_i} = -\infty \tag{8}$$

$$\lim_{x_i \to \infty} M(x_i, x_j) = 0 \qquad , \text{for } i = 1, 2 \tag{9}$$

$$\lim_{x_i \to \infty} \frac{\partial M(x_i, x_j)}{\partial x_i} = 0 \qquad , \text{for } i = 1, 2 \tag{10}$$

$$\tag{11}$$

This implies that achieving "infinite" product differentiation ($\mu \to 0$) is infinitely costly and if firms make no investments, their products become perfect substitutes.

3.2 Dynamic investment in product innovation

In each period t, firms choose prices $p_{i,t}$ and investment levels $x_{i,t}$. As in the benchmark model, firms' profits at time t depend on prices $(p_{i,t}, p_{j,t})$ and the state variable (μ_t). Investments aim at increasing product differentiation (lowering the state variable μ_t). At the end of the period, the stochastic outcome of the investment decision is realized.

For practical reasons, I will assume that the state variable takes on values in the non-negative integers, $\mu_t = \{0, 1, 2, \dots, M-1\}$. The level of product homogeneity evolves over time with an exogenous process Δ_t and the outcome of the firms' investments η_t. Δ_t captures a convergence to product homogeneity. If firms make no investments, over time there are spillover effects of innovation and products will become more similar. Thus,

$$\mu_{t+1} = \mu_t - \eta_{t+1} + \Delta_{t+1} \tag{12}$$

As in Fershtman & Pakes (2000), I will consider equilibria that depend on the pay-off relevant state variable μ_t and a set of indicator functions that keep track of past deviations. Assume that $\alpha_i \in \{0, 1\}$ which of the firms has deviated from the collusive agreement in the past ($\alpha_i = 1$). This indicator can be interpreted as a firm's reputation or trustworthiness (if $\alpha_i = 0$, firm i is trustworthy). Thus, price and investment strategies become functions of the state variables: $p_{i,t}(\mu_t, \alpha_{i,t}, \alpha_{j,t})$.

Assume the same demand structure as in the benchmark model. Recall that Nash equilibrium, collusive and deviation profits, respectively, are:

$$\Pi_t^{NE}(\mu_t) = \frac{(2+\mu_t)v^2}{(4+\mu_t)^2}$$
$$\Pi_t^{C}(\mu_t) = \frac{v^2}{8}$$
$$\Pi_t^{D}(\mu_t) = \frac{(4+\mu_t)^2 v^2}{64(2+\mu_t)}$$

4 Preliminary results

4.1 Benchmark: Static investment decision

4.1.1 Collusive equilibrium

With collusion, firms solve the joint maximization problem in (5). This his yields the following symmetric equilibrium prices, quantities and profits:

$$p^C = \frac{v}{2} \tag{13}$$
$$q^C = \frac{v}{4} \tag{14}$$
$$\Pi^C = \frac{v^2}{8} \tag{15}$$

4.1.2 Static Nash equilibrium

Without collusion, firms solve the maximization problem in (4). This yields the following symmetric equilibrium prices, quantities and profits:

$$p^{NE} = \frac{2v}{(4+\mu)} \tag{16}$$
$$q^{NE} = \frac{(2+\mu)v}{2(4+\mu)} \tag{17}$$
$$\Pi^{NE} = \frac{(2+\mu)v^2}{(4+\mu)^2} \tag{18}$$

Note that as products become perfectly homogeneous ($\mu \to \infty$), the Nash equilibrium price approaches marginal cost (which is equal to zero). When products become completely differentiated ($\mu \to 0$), the Nash equilibrium price approaches the collusive price:

$$\lim_{\mu \to \infty} p^{NE} = 0$$
$$\lim_{\mu \to 0} p^{NE} = \frac{v}{2} = p^C$$

4.1.3 Deviation

If one firm deviates from the collusive agreement, it chooses a price to maximize its profit. In the deviation period, the opponent cannot respond to to the deviation and sets the collusive price p^C. Thus, the deviating firm solves:

$$\max_{p_i} (p_i - c) q_i(p_i, p^C) = (p_i - c) \frac{1}{2} \left[v - \frac{(2+\mu)}{2} p_i + \frac{\mu}{2} p^C \right], \tag{19}$$

where p^C is given in (13). This his yields the following deviation price, quantity and profit for the deviating firm:

$$p^D = \frac{(4+\mu)v}{4(2+\mu)} \tag{20}$$
$$q^D = \frac{(4+\mu)v}{16} \tag{21}$$
$$\Pi^D = \frac{(4+\mu)^2 v^2}{64(2+\mu)} \tag{22}$$

4.1.4 Incentive compatibility

For the joint profit maximum to be sustainable in equilibrium, the following condition must hold:

$$\frac{\Pi^C}{1-\delta} \geq \Pi^D + \frac{\delta \Pi^{NE}}{1-\delta}$$
$$\Updownarrow$$
$$\delta \geq \frac{\Pi^D - \Pi^C}{\Pi^D - \Pi^{NE}} \equiv \underline{\delta}, \tag{23}$$

where δ is the firms' common discount factor and $\underline{\delta}$ denotes the critical value of patience for collusion to be sustainable.

Intuitively, product differentiation affects the cartel stability in two ways. When products become more differentiated, Nash equilibrium profits increase. The punishment for deviation (the return to Nash equilibrium) therefore becomes less severe. This effect makes collusion harder to sustain. On the other hand, the deviating firm can steal less demand from its opponent when products are more differentiated. This makes deviation less profitable and collusion easier to sustain. In this model, the latter effect dominates. To see this, note that the critical discount factor $\underline{\delta}$ is strictly decreasing in product differentiation (recall that a smaller μ implies more differentiated products):

$$\begin{aligned} -\frac{d\underline{\delta}}{d\mu} &= -\frac{d}{d\mu}\left[\frac{(\mu+4)^2}{\mu^2+16\mu+32}\right] \\ -\frac{d\underline{\delta}}{d\mu} &= -\frac{8\mu(\mu+4)}{(\mu^2+16\mu+32)^2} < 0 \qquad , \text{for } \mu \geq 0 \end{aligned} \tag{24}$$

The more differentiated the products (the smaller μ), the smaller is the critical discount factor $\underline{\delta}$ and collusion becomes easier to sustain. Note that when products become perfect substitutes, the incentive to deviate becomes so large that collusion is not sustainable:[1]

$$\lim_{\mu\to\infty} \underline{\delta} = \lim_{\mu\to\infty} \frac{(\mu+4)^2}{\mu^2+16\mu+32} = 1 \tag{25}$$

4.1.5 Investment choice

Without collusion

Without collusion, firms choose investment levels to maximize the net present value of future (Nash equilibrium) profits net of investment cost x_i:

$$\max_{x_i} \frac{\Pi^{NE}\big(\mu(x_i, x_j)\big)}{1-\delta} - x_i \tag{26}$$

The first-order condition yields:

$$\begin{aligned} \frac{1}{1-\delta}\frac{d\Pi^{NE}}{d\mu}\frac{\partial M}{\partial x_i} &= 1 \\ -\frac{1}{1-\delta}\frac{d\Pi^{NE}}{d\mu} &= -\frac{1}{\frac{\partial M}{\partial x_i}} \\ 0 < \frac{1}{1-\delta}\frac{M(x_i, x_j)v^2}{(4+M(x_i, x_j))^3} &= -\frac{\partial x_i}{\partial \mu} \end{aligned} \tag{27}$$

Thus, firms weigh the cost of investment against their future profit gains from increased product differentiation. Without collusion, firms choose positive investment levels. Equation (27) implicitly defines firm i's optimal investment level as a function of the firms' patience δ and its opponent's investment x_j. An analogous expression can be

[1] In this analysis, I need to include a non-negativity constraint for the non-deviating firm's quantity. For Bertrand competition with differentiated products, the critical discount factor is $\delta \geq \underline{\delta} = 1 - \frac{1}{N}$, where N is the number of firms in the industry. For a duopoly with homogeneous products, the critical discount factor should converge to $\frac{1}{2}$. I suspect that the results below do not hold when including a non-negativity constraint for quantities.

found for firm j. Denote the symmetric optimal investment level without collusion by x^{NE}.

Note that as firms become more patient, the left-hand side of (27) increases. This implies that the marginal investment cost must increase too. As investment costs are assumed to be convex, this implies that firms invest more in innovation, the more patient they are: $\frac{dx^{NE}}{d\delta} > 0$.

With collusion

As firms do not collude on investment, their maximization problem is similar to (26):

$$\max_{x_i} \frac{\Pi^C\big(\mu(x_i, x_j)\big)}{1-\delta} - x_i \tag{28}$$

The first-order condition yields:

$$\begin{aligned} \frac{1}{1-\delta}\frac{d\Pi^C}{d\mu}\frac{\partial M}{\partial x_i} &= 1 \\ -\frac{1}{1-\delta}\frac{d\Pi^C}{d\mu} &= -\frac{1}{\frac{\partial M}{\partial x_i}} \\ 0 &= -\frac{1}{\frac{\partial M}{\partial x_i}} \end{aligned} \tag{29}$$

Given the asymptotic property of $M(x_i, x_j)$ given in (8), the symmetric solution to the investment problem is $x^C = 0$. Thus, collusion removes the firms' incentives to invest. This model may therefore offer an additional explanation to why cartels are often found in markets with relatively homogeneous products: Cartelization reduces incentives to soften price competition – and therefore incentives to invest in product innovation and differentiation.

In this model, no investment implies that products become homogeneous. When products are homogeneous, $\underline{\delta} \to 1$ (according to (25)) and the incentive compatibility constraint is violated. Thus, firms must invest at least:[2]

$$x^C = \arg\min_x x \qquad \text{s.t. } \delta \geq \frac{(M(x,x)+4)^2}{M(x,x)^2 + 16M(x,x) + 32} = \underline{\delta}\big(M(x,x)\big) \tag{30}$$

The right-hand side of the inequality is decreasing in the firms' investments x.[3] Thus,

[2]Instead of increasing investments, firms could set collusive prices below the joint profit maximizing price. With collusion, firms will have to make a tradeoff between investment costs and reduced collusive prices. Rather than assuming that they adjust investment to achieve incentive compatibility, I would like to model this trade-off explicitly.

[3]Note that $\frac{d\delta}{dx_i} = \frac{d\delta}{d\mu}\frac{\partial M}{\partial x_i}$, where $\frac{d\delta}{d\mu} > 0$ and $\frac{\partial M}{\partial x_i} < 0$.

$x^C(\delta)$ is implicitly defined by:

$$\delta = \frac{(M(x^C(\delta), x^C(\delta)) + 4)^2}{M(x^C(\delta), x^C(\delta))^2 + 16M(x^C(\delta), x^C(\delta)) + 32},$$

and $\frac{dx^C}{d\delta} < 0$.

Whether firms invest less with collusion, depends on the firms' patience δ. When firms are very patient, products can be very homogeneous (μ can be large) without violating the incentive compatibility constraint. This implies that investments can be small. Conversely, when firms' patience is low ($\delta \to \frac{1}{2}$), products must be very differentiated to sustain collusion. This implies that investments must be large:

$$\delta \to 1 \Rightarrow \mu \to \infty \Rightarrow x^C \to 0$$
$$\delta \to \frac{1}{2} \Rightarrow \mu \to 0 \Rightarrow x^C \to \infty$$

If firms are sufficiently patient, they invest less in product differentiation if the industry is cartelized. Intuitively, firms that compete will be more willing to invest in product differentiation if they are patient, as they are more willing to pay for an investment today to soften long-run price competition. For a cartelized industry, patient firms imply that collusion is easier to sustain. As firms must invest to disincentivise deviation (and lower the critical discount factor), high patience implies that collusion can be sustained at lower investment levels. Similar to the findings of Friedman & Thisse (1993), I find that partial collusion fosters less investment (and less product differentiation) if firms are sufficiently patient. However, if firms are sufficiently impatient, the result is reversed.

Fershtman & Gandal (1994) show that, firms would prefer to commit to no collusion. As collusion intensifies along non-price dimensions, firms' profits are reduced. In the repeated price game, collusion is always beneficial ($\Pi^C > \Pi^{NE}$). Firms therefore summer from a commitment problem. In my model, this commitment problem does not exists. Firms prefer non-collusion only if collusion requires a large investment today (firms are sufficiently impatient):

$$\frac{\Pi^{NE}}{1-\delta} - x^{NE} > \frac{\Pi^C}{1-\delta} - x^C$$
$$\Updownarrow$$
$$x^C - x^{NE} > \frac{\Pi^C - \Pi^{NE}}{1-\delta} \geq 0 \tag{31}$$

Thus, if firms choose a low investment level $x^{NE} < x^C$ in the first stage, the incentive compatibility constraint is violated and collusion cannot be sustained. Consequently, firms can commit to compete in the price game.[4]

4.2 Dynamic investment in product innovation

Let $\rho(\mu'|\mu, x, \alpha)$ denote the conditional probability that $\mu_{t+1} = \mu'$ given current state variables μ and α, and investment x. The Bellman equation is:

$$V(\mu,\alpha) = \Pi(\mu,\alpha) + \max_{x\geq 0}\left[-x + \delta\sum_{\mu'} V(\mu',\alpha')\rho(\mu'|\mu,x,\alpha)\right], \tag{32}$$

where

$$\Pi(\mu,\alpha) = \mathbf{I}(\mu,\alpha)\Pi^C + \big(1-\mathbf{I}(\mu,\alpha)\big)\Pi^{NE}(\mu)$$

and

$\mathbf{I}(\mu,\alpha) \in \{0,1\}$, where $\mathbf{I}(\mu,\alpha) = 1$ iff:

i) $\alpha = 0$, and

ii) for $i = 1, 2$
$\Pi^C + \max_{x\geq 0}\left[-x + \delta\sum_{\mu'} V(\mu',\alpha')\rho(\mu'|\mu,x,\alpha=0)\right]$
$> \Pi^D(\mu) + \max_{x\geq 0}\left[-x + \delta\sum_{\mu'} V(\mu',\alpha')\rho(\mu'|\mu,x,\alpha_i=1,\alpha_j=0)\right]$

The Bellman equation states that a firm's continuation value is equal to its current period profit plus the discounted expected value of future returns. Its current period profit is equal to the collusive profit ($\mathbf{I}(\mu,\alpha) = 1$) if no firm has deviated in the past ($\alpha = 0$) and no firm has an incentive to deviate from the collusive agreement.

In future work with this project, I wish to find numerical results for equilibrium investment, realized level of product differentiation, prices, and consumer surplus with and without collusion. I would also like to compare these to the static model. The results can be represented in a similar table as in Fershtman & Pakes (2000) welfare analysis. This remains a work in progress.

[4]Note that this result depends on the assumption that cartel members set prices to maximize joint profit. Instead of increasing investments to ensure that the incentive compatibility constraint holds, firms could set collusive prices below the joint profit-maximizing price. This could reintroduce the commitment problem. As a reduction in the collusive price also lowers the necessary investment level, it may no longer be the case that firms strictly prefer competition to collusion (the condition in (31) may be violated). This requires further analysis.

5 Work remaining

Some work remains in solving the baseline model. In the analysis of the incentive compatibility constraint, I need to include a non-negativity constraint for the non-deviating firm's quantity. For Bertrand competition with differentiated products, the critical discount factor is $\delta \geq \underline{\delta} = 1 - \frac{1}{N}$, where N is the number of firms in the industry. For a duopoly with homogeneous products, the critical discount factor should converge to $\frac{1}{2}$. I suspect that the results below do not hold when including a non-negativity constraint for quantities.

The result that there is no commitment problem depends on the assumption that cartel members set prices to maximize joint profit. Instead of increasing investments, firms could set collusive prices below the joint profit maximizing price. With collusion, firms will have to make a tradeoff between investment costs and reduced collusive prices. Rather than assuming that they adjust investment to achieve incentive compatibility, I would like to model this trade-off explicitly. This could reintroduce the commitment problem. As a reduction in the collusive price also lowers the necessary investment level, it may no longer be the case that firms strictly prefer competition to collusion (the condition in (31) may be violated). This requires further analysis.

The bulk of work lies in solving the dynamic model. I wish to find numerical results and compare the outcomes with and without collusion in both models. The results can be represented in a similar table as in Fershtman & Pakes (2000) welfare analysis.

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